3/16/17

"Life is 10% what happens to me and 90% of how I react to it."-Charles Swindoll

HW: "Finding Equations of Exponential Functions" homework

AIM: How do we write Equations of Exponential Functions?

Warm Up:

1. If $f^{-1}(x) = 4x + 3$ then which of the following is the correct formula for $f(x)$?

(a) $f(x) = \frac{1}{4}x - 3$

(b) $f(x) = \frac{1}{4}x - \frac{3}{4}$

(c) $f(x) = -4x - 3$

(d) $f(x) = 4x - 3$

$y = 4x + 3$

$x = 4y + 3$

$x - 3 = 4y$

$\frac{1}{4}x - \frac{3}{4} = y$

$f(-3) = 7$

2. The function $f(x)$ is an even function with $f(3) = 7$ and $f(9) = 11$. What is the average rate of change of $f(x)$ over the interval $-3 \le x \le 9$?

a b

$$\frac{f(9) - f(-3)}{9 - (-3)} = \frac{11 - 7}{9 + 3} = \frac{4}{12} = \frac{1}{3}$$

1. Which of the following represents an exponential function?

(1) $y=3x-7$ (3) $y=3(7)^x$

(2) $y=7x^3$ (4) $y=3x^2+7$

An exponential function must have a base that is constant and an exponent that is variable.

(3)

2. If $f(x)=6(9)^x$ then $f\left(\frac{1}{2}\right)=?$ (Remember what we just learned about fractional exponents and do without a calculator.)

(1) $\frac{7}{2}$ (3) 27

(2) 18 (4) $\frac{15}{2}$

$f\left(\frac{1}{2}\right)=6(9)^{\frac{1}{2}}=6\left(\sqrt{9}\right)=6(3)=18$

(2)

3. If $h(x)=3^x$ and $g(x)=5x-7$ then $h(g(2))=$

(1) 18 (3) 38

(2) 12 (4) 27

$g(2)=5(2)-7=3$

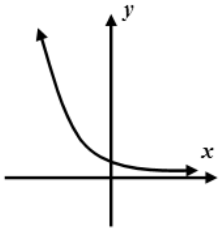
$h(3)=3^3=27$

(4)

4. Which of the following equations could describe the graph shown below?

(1) $\cancel{y=x^2+1}$ (3) $\cancel{y=-2x+1}$

(2) $y=\left(\frac{2}{3}\right)^x$ (4) $y=4^x$



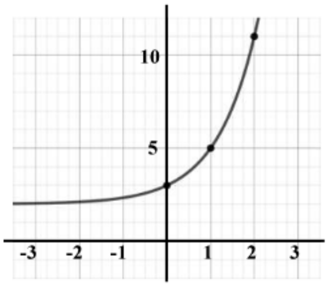
(2)

5. Which of the following equations represents the graph shown?

(1) $y=5^x$ (3) $\cancel{y=\left(\frac{1}{2}\right)^x+2}$

(2) $y=4^x+1$ (4) $y=3^x+2$

See which of these four equations has the points (0, 3), (1, 5), and (2, 11) lie on its graph.



(4)

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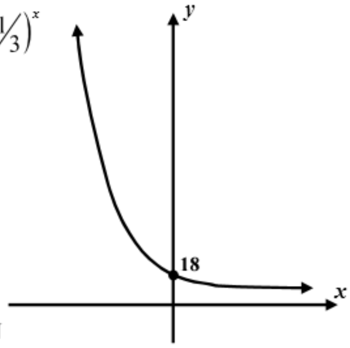
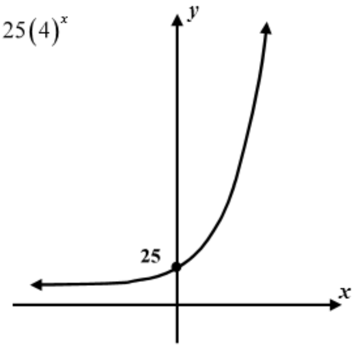
6. Sketch graphs of the equations shown below on the axes given. Label the y-intercepts of each graph.

(a) $y = 18\left(\frac{1}{3}\right)^x$



(b) $y = 25(4)^x$



APPLICATION

7. The Fahrenheit temperature of a cup of coffee, F, starts at a temperature of 185°F. It cools down according to the exponential function $F(m) = 113\left(\frac{1}{2}\right)^{m/20} + 72$, where m is the number minutes it has been cooling.

(a) How do you interpret the statement that $F(60) = 86$?

If I go 60 min the temp will be 86°

After 60 minutes, the coffee is at a temperature of 86 degrees Fahrenheit.

(b) Determine the temperature of the coffee after one day using your calculator. What do you think this temperature represents about the physical situation?

The temp will stop decreasing when it reaches room temp

1 day = 24 ~~hrs~~ × $\frac{60\text{ min}}{\cancel{\text{hr}}}$ = 1440 min

$$F(1440) = 113\left(\frac{1}{2}\right)^{1440/\cancel{20}} + 72$$

$$= 72$$

∴ 72° = room temperature

It represents the temperature of the room the coffee is cooling down in.

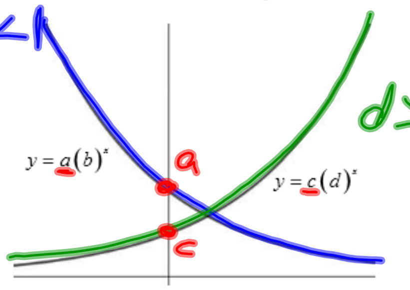
REASONING

8. The graph below shows two exponential functions, with real number constants a, b, c, and d. Given the graphs, only one pair of the constants shown below could be equal in value. Determine which pair could be equal and explain your reasoning.

~~b and d~~ a and b ~~a and c~~

$0 < b < 1$

$d > 1$



check out bases

b and d cannot be equal because b is less than 1 and d is greater than 1.

check out y-intercepts

a and c cannot be equal because they are the y-intercepts of the two graphs and clearly aren't the same.

Default if other 2 do not work

a and b could be equal. Although we know b is less than 1, there is not reason a could not be less than 1 as well.

9. Explain why the equation below can have no real solutions. If you need to, graph both sides of the equation using your calculator to visualize the reason.

$$3^x + 5 = 2$$
$$-5 \quad -5$$

$$3^x = -3$$

If $3^x + 5 = 2$ then $3^x = -3$.
But we cannot raise a positive number to any power and produce a negative.
Hence, this equation has no solutions.

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One of the skills that you acquired in Common Core Algebra I was the ability to write equations of exponential functions if you had information about the starting value and base (multiplier or growth constant). Let's review a very basic problem.

* y-intercept

Exercise #1: An exponential function of the form $f(x) = a(b)^x$ is presented in the table below. Determine the values of a and b and explain your reasoning.

$a =$ 5 ← the y-int.

$b =$ 3

Final Equation: $f(x) = 5(3)^x$

x	0	1	2	3
$f(x)$	5	15	45	135

×3 ×3 ×3

Explanation: 3 is the multiplier.

a is the y-int

b is the multiplier (base)

Finding an exponential equation becomes much more challenging if we do not have output values for inputs that are increasing by unit values (increasing by 1 unit at a time). Let's start with a basic problem.

y-int: (0,8)

Exercise #2: For an exponential function of the form $f(x) = a(b)^x$, it is known that $f(0) = 8$ and $\boxed{f(3) = 1000}$.

(0,8) (3,1,000)

(a) Use the fact that $f(0) = 8$ to determine the value of a. Show your thinking.

f(0)=8 gives us the point (0,8) which is the y-int

$a = 8$

(b) Use you answer from part (a) and the fact that $f(3) = 1000$ to set up an equation to solve for b. You will solve for b in part (c).

$f(x) = 8b^x$ ← Plug it in

$1000 = 8b^3$

(c) Solve for the value of b using properties of exponents.

$\frac{1000}{8} = \frac{8b^3}{8}$

$125 = b^3$

$\sqrt[3]{125} = b$

$b = 5$

(d) Determine the value of $f(2)$

Equation:

$f(x) = 8(5)^x$

$f(2) = 8(5)^2$

$f(2) = 8(25)$

$\boxed{f(2) = 200}$

Exercise #3: An exponential function exists such that $f(4)=3$ and $f(6)=48$, which of the following must be the value of its base? Explain or illustrate your thinking.

(1) $b=16$ (3) $b=6$

(2) $b=2$ (4) $b=4$

x	4	5	6
f(x)	3		48

$b \cdot b = b^2$

$$\frac{3b^2}{3}=\frac{48}{3}$$

$$b^2=16 \longrightarrow b=4$$

Now, let's work with the most generic type of problem. Just like with lines, **any two (non-vertically aligned) points will uniquely determine the equation of an exponential function**.

Exercise #4: An exponential function of the form $y=a(b)^x$ passes through the points $(2,36)$ and $(5,121.5)$.

(a) By substituting these two points into the general form of the exponential, create a **system of equations** in the constants a and b.

$$36=a(b)^2$$
$$121.5=a(b)^5$$

(b) Divide these two equations to eliminate the constant a. Recall that when dividing to like bases, you subtract their exponents.

$$\frac{36}{121.5}=\frac{\cancel{a}\cancel{b^2}}{\cancel{a}\cancel{b^5}_{b^3}}$$

$$\frac{36}{121.5}=\frac{1}{b^3}$$

(c) Solve the resulting equation from (b) for the base, b.

$$\frac{36}{121.5}=\frac{1}{b^3}$$

$$\frac{121.5}{36}=\frac{36b^3}{36}$$

$$3.375=b^3$$

$$b=1.5$$

(d) Use your value from (c) to determine the value of a. State the final equation.

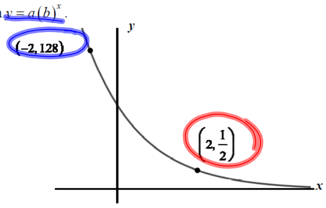
$b=1.5$ use either point $(5,121.5)$

$$y=ab^x$$
$$121.5=a(1.5)^5$$
$$\frac{121.5}{7.59375}=\frac{a(7.59375)}{7.59375}$$
$$a=16$$

Equation:
$$y=16(1.5)^x$$

Let's now get some practice on this with a decreasing exponential function.

Exercise #5: Find the equation of the exponential function shown graphed below. Be careful in terms of your exponent manipulation. State your final answer in the form $y=a(b)^x$.



$$128=a(b)^{-2}$$
$$.5=a(b)^2$$

$$\frac{128}{.5}=\frac{\cancel{a}\cancel{b^{-2}}}{\cancel{a}\cancel{b^2}_{b^4}}$$

$$\frac{256}{1}=\frac{1}{b^4}$$

$$\frac{256b^4}{256}=\frac{1}{256}$$

$$\sqrt[4]{b^4}=\sqrt[4]{\frac{1}{256}}$$

$$b=.25=\frac{1}{4}$$

Find a:

$b=\frac{1}{4}$ use $(-2,128)$

$$y=ab^x$$
$$128=a\left(\tfrac{1}{4}\right)^{-2}$$
$$\frac{128}{16}=\frac{a(16)}{16}$$
$$a=8$$

$$y=8\left(\tfrac{1}{4}\right)^x$$

***Exercise* #6:** A bacterial colony is growing at an exponential rate. It is known that after 4 hours, its population is at 98 bacteria and after 9 hours it is 189 bacteria. Determine an equation in $y = a(b)^x$ form that models the population, y, as a function of the number of hours, x. At what percent rate is the population growing per hour?

$(4, 98)$

$(9, 189)$

$\frac{98}{189} = \frac{\cancel{a}b^4}{\cancel{a}b^9}$

$\frac{98}{189} = \frac{1}{b^5}$

$98b^5 = 189$

$b^5 = \frac{189}{98}$

$\frac{189}{98} = \frac{\cancel{a}\cancel{b^9}^{\,b^5}}{\cancel{a}\cancel{b^4}}$

$\sqrt[5]{\frac{189}{98}} = \sqrt[5]{b^5}$

$1.14 = b$

Find a:

$b = 1.14 \quad (4, 98)$

$y = ab^x$

$98 = a(1.14)^4$

$98 = a(1.68896016)$

$a \approx 58$

Equation:

$$y = 58(1.14)^x$$

increase

.14 is 14%

1) (0, 5) ← y-int gives us "a"

(3, 320)

$y = ab^x$

$a = 5$

To find b:

$\frac{320}{5} = \frac{5 \cdot b^3}{5}$

$\frac{320}{5} = b^3$

$64 = b^3$

$b = 4$

$$\boxed{y = 5(4)^x}$$